WP1.6 – DESIGN OF STIFFENING RINGS

BACKGROUND DOCUMENT
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1. **WORK PACKAGE DESCRIPTION**

WP leader: AUTH

Contractors: GLWIND

**Task:** Review of the best practice for design of stiffening rings.

**Deliverables:** Background document

**Starts:** 01/01/2007

**Ends:** 31/03/2007
2. BACKGROUND DOCUMENT

2.1. General

By definition, ring stiffeners are local stiffening members that pass around the circumference of the shell of revolution at a given point on the meridian. Normally they are attached to the interior of the shell of the wind turbine tower and are formed as single plated sections (see Figure F-2.1); "T" or "L" profiles are not common in wind towers. The rings are assumed to have limited stiffness for deformations out of their own plane (meridional displacements of the shell) but they should be stiff for deformations in the plane of the ring.

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Figure F-2.1-1: Typical stiffening ring cross-section

The beneficial impact of the stiffening rings to the overall performance of the shell, when the Plastic Limit State (LS1) is investigated, is restricted to the flanges mainly, which are relieved from the excessive strain otherwise induced by the concentrated
circumferential stresses (see Figure F-2.1-6). In contradiction, the stiffened and the unstiffened shell appear to respond to the applied loading in a comparable mode, the differences between the resultant Von Mises stresses being trivial (see Figure F-2.1-3).

As regards the Buckling Limit State (LS3), the combination of two factors:

- The distribution of the wind pressure, as determined for example by [EC 1-1-4 §7.9] in analytical form, when introduced to the structural model (see Figure F-2.1-2) tends to "ovalise" the circular section, imposing consequently quite high circumferential stresses (see Figure F-2.1-5).

- The circumferential buckling strength of the shell is limited, due to the small thickness to radius ratio of the tower shell. In this case, the contribution of the rings is proved to be determinative, since both the above problems are met: The maximum circumferential stresses are reduced noticeably (see Figure F-2.1-5) and at the same time the buckling strength is increased as much as ten times. It is the designer's and the manufacturers job then the definition of the most cost effective configuration to be adopted: The unstiffened shell with thicker plates of the stiffened one, with plates of reduced thickness.

The local buckling assessment can be based on the provisions of [EC 3-1-6] and specifically following either the global numerical analyses of [§8.6] and [§8.7], or the stress design procedure of [§8.5]. The engineer though must be aware of the fact that the later method leads inevitably to the installation of stiffening rings because, as derived by the formulas of Annex [D] in the case of long cylinders:

$$\sigma_{\theta,Rcr} = E \cdot \left( \frac{t}{r} \right)^2 \cdot \left[ 0.275 + 2.03 \cdot \left( \frac{C_{\theta}}{\ell} \right)^4 \cdot \frac{\ell}{r^2} \right]$$

the critical circumferential buckling stress [$\sigma_{\theta,Rcr}$] is diminished for a given significant magnitude of the length [\ell] between boundaries (flanges, stiffening rings or base support), resulting thus in unacceptable shell plate thicknesses.
Figure F-2.1-3: Shell – Von Mises Stresses
Figure F-2.1-4: Shell – Meridional Stresses
Figure F-2.1-5: Shell – Circumferential Stresses
Figure F-2.1-6: Flanges – Von Mises Stresses

Stiffened shell  Unstiffened shell
2.2. Ring design

Stiffening rings should be designed so that the shell buckling is restricted within the length of adjusted ring (see Figure F-2.2-1). It is desirable to provide rings with sufficient residual strength to prevent general instability. However, the rings may rotate or warp out of their plane. Local instabilities reduce their capacity.

The ring should be checked for:

- Resistance to plastic limit under circumferential compression;
- Resistance to buckling under circumferential compression;
- Resistance to local yielding under tension or compression stresses;
- Resistance to torsion;
- Resistance of joints (connections).

The recommended approach for the design is always the F.E. numerical analysis, by means of a global model including all the tower parts and the appropriate boundary

Figure F-2.2-1: 1st buckling mode (CLF=Critical Load Factor)
conditions. Load effects may be determined by assuming relevant plastic collapse mechanisms. The characteristic resistance shall be determined by recognized methods of plastic theory. In this case, the selection of the most cost-effective cross section of the rings and the distance between the successive elements can be determined with satisfactory accuracy and affordable effort (see Figure F-2.2-2). When the geometrically non-linear elastic analysis with imperfections (GNIA) is adopted, the verification of the ring is direct, otherwise the corresponding to the specific type of analysis supplementary checks, mainly against circumferential buckling, should be carried out.

2.3. Code specifications

For the preliminary dimensioning of the ring stiffeners or when a more thorough assessment using numerical analysis is not implemented, the provisions of the Codes concerning both strength and stiffness requirements should be obeyed. A brief presentation of the relevant clauses is as follows:
2.3.1 General geometry requirements

To prevent local buckling of ring as a possible failure mode the height to thickness ratio should be restricted within the limits:

\[
\frac{h}{t} \leq 10 \cdot \frac{235}{f_y} \quad \text{[EC 3-1-1 Table 5.2] class [2] cross section requirements}
\]

\[
\frac{h}{t} \leq 14 \cdot \frac{235}{f_y} \quad \text{[EC 3-1-1 Table 5.2] class [3] cross section requirements}
\]

\[
\frac{h}{t} \leq 12 \cdot \frac{235}{f_y} \quad \text{NORSOK standard [N-004 §6.3.6.2]}
\]

Where membrane theory is used to find the primary stresses in the shell, discrete rings attached to an isotropic cylindrical silo shell may be deemed to have an effective area which includes a length of shell above and below the ring of: 0.78rt.

2.3.2 Stiffness requirements – Circumferential buckling: [EC 3-4-1 §5.3.2.5]

The flexural rigidity \([E I_z]\) of a ring at the upper edge of the cylinder about its vertical axis (circumferential bending) should satisfy both conditions:

\[
E \cdot I_z \geq k_1 \cdot E \cdot L \cdot t^3
\]

\[
E \cdot I_z \geq 0,08 \cdot C_w \cdot E \cdot r \cdot t^3 \cdot \sqrt{r/t}
\]

where:

- \(r\) : Radius of shell middle surface
- \(I_z\) : Second moment of area of the ring for circumferential bending
- \(E\) : Modulus of elasticity of steel
- \(L\) : Total height of the shell wall
- \(C_w\) : Wind pressure distribution coefficient, given by \([§5.3.2.5 (6)÷(9)]\) of the Code
- \(k_1\) : Constant determined by the National Annex. Recommended value: \(k_1 = 0,10\)
- \(t\) : Thickness of the thinnest strake

2.3.3 Stiffness requirements – Shear buckling: [EC 3-4-1 §5.3.2.6]

A stiffening ring which is required as the boundary for a shear buckling zone should have a flexural rigidity \([E I_z]\) about the axis for bending around the circumference not less than:
where:

\[ E \cdot I \geq k \cdot E \cdot t^3 \cdot \sqrt{r} \cdot l \]

\begin{align*}
E, r, t & : \text{As defined in } [\S 2.3.2] \\
l & : \text{Height between stiffening rings or boundaries} \\
k_s & : \text{Constant determined by the National Annex. Recommended value: } k_s = 0.10
\end{align*}

### 2.3.4 Stiffness requirements: [NORSOK Standard N-004 §6.3.6]

The requirements given in this section apply to tubulars having a thickness \( t \geq 6\text{mm} \) and \( D/t < 120 \). The circumferential stiffening ring size may be selected on the following approximate basis:

\[ I = C_h \cdot \frac{t^2 \cdot L_r \cdot D}{4} \]

where:

\begin{align*}
I_c & : \text{Required moment of inertia for ring composite section} \\
L_r & : \text{Ring spacing} \\
D & : \text{Diameter of the shell} \\
C_h & : \text{Parameter, which can be approximated, for a typical wind tower shell, as:}
\]

\[ C_h \approx 0.76 \left( 2 \cdot \frac{l}{D \cdot t} - 0.58 \right) \]

An effective width of shell equal to \( 1.1 \cdot \sqrt{D \cdot t} \) may be assumed as the flange for the composite ring section.

### 2.3.5 Spacing requirements: [EC 3-4-1 §5.3.2.6]

The critical buckling external pressure for an isotropic wall should be found as:

\[ p_{n,Rcr} = 0.92 \cdot C_b \cdot C_w \cdot E \cdot \left( \frac{r}{l} \right) \cdot \left( \frac{t}{r} \right)^{2.5} \]

where:

\begin{align*}
t & : \text{Thickness of the thinnest part of the wall} \\
l & : \text{Height between stiffening rings or boundaries} \\
p_{n,Rcr} & : \text{Pressure difference between the outside and inside; positive when acting inwards} \\
C_b & : \text{External pressure buckling coefficient. In this case: } C_b = 1.00 \\
C_w & : \text{Wind pressure distribution coefficient, as a function of } [C_b], [r] \text{ & } [l]: \ C_w \geq 1.00
\end{align*}
2.3.6 Strength requirements: [EC 3-4-1 §5.3.2.5]

For the upper edge of a cylinder to be treated as effectively restrained by a ring, the design value of the circumferential (hoop) force and circumferential bending moment about a vertical axis in the ring should be taken as:

\[ N_{\theta,Ed} = 0.5 \cdot r \cdot L \cdot p_{n,Ed} \]
\[ M_{\theta,Ed} = M_{\theta,Edo} + M_{\theta,Edw} \]

with:

\[ M_{\theta,Edo} = 0.0033 \cdot p_{n,S1} \cdot r^2 \cdot L \cdot \left( \frac{P_{n,S1}}{P_{n,S1} - P_{n,Edu}} \right) \]
\[ M_{\theta,Edw} = 0.17 \cdot p_{n,Edw} \cdot r^2 \cdot L \cdot \left( \frac{P_{n,Edw}}{P_{n,S1} - P_{n,Edu}} \right) \]

\[ p_{n,S1} = \frac{6 \cdot z \cdot E \cdot I_z}{r^3 \cdot L} \]

where:

- \( p_{n,Ed} \): Design value of the maximum external pressure under wind or partial vacuum
- \( p_{n,Edu} \): Design value of the uniform component of the external pressure
- \( p_{n,Edw} \): Design value of the stagnation point pressure under wind
- \( p_{n,S1} \): Reference pressure for ring bending moment evaluations
- \( M_{\theta,Edo} \): Design value of the bending moment associated with out-of-roundness; it should be increased by 15%, if the ring is made as a cold formed construction
- \( M_{\theta,Edw} \): Design value of the bending moment due to wind
- \( L,E,r,t \): As defined in §2.3.2
2.4. References